Information transmission over a finite buffer channel

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Abstract — **We study information transmission through a finite buffer channel modeled as a concatenation of a discrete memoryless channel and a finite state erasure channel. The state of the erasure channel is determined by the buffer occupancy upon arrival of the transmission symbol; an erasure occurs when an input arrives to a full buffer. We show that the capacity of the channel depends on the long-term loss probability of the buffer and the capacity of the DMC. Thus, even though the channel itself has memory, the capacity apparently depends only on the stationary loss probability of the buffer. We also show that delayed feedback does not help in this channel. We also study the channel as a deletion channel where we do not know where the erasures have occurred.**

I. Summary

We propose a channel abstraction for the finite-buffer channel and study its capacity. This model is motivated by packet-switched networks, where a packet is queued in a finite buffer on each router along its path through the network. A packet can be dropped because of buffer overflow, or corrupted due to transmission errors. We *do not* consider coding in inter-arrival times in this abstraction[1]. Note that the sender may have control over the long-term packet arrival rate, which affects the loss process at the buffer; however, there is no side information transmitted using the arrival process.

We formulate this problem as transmission over a finite state channel where the transitions of the finite state channel occur due to arrivals and departures of packets to the buffer. The model considered resembles the problem of transmission through finite state channels studied extensively [2]. But one of the differences is that the state process need not be Markovian (see Figure I). In this paper we consider only a single user's packets arriving at the buffer and the buffer state is affected by the arrivals of that user.

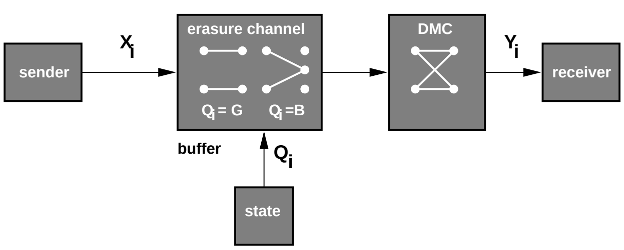

Figure 1: Finite-state channel model.

We first consider the problem where the receiver knows when a packet is dropped. In practice, this is done using a sequence number associated with packets. Later we study the channel where this is not known and model it as a deletion channel. Under regularity conditions on the state transition process we can prove a coding theorem for the proposed channel model [3]. We show that though this channel has memory, the capacity is determined by the long term stationary loss probability of the buffer. That is, the capacity is the product of the capacity of the DMC and that of the long term probability of a packet getting through. This shows that even though the finite buffer channel has complicated memory, its capacity behavior is akin to a simple erasure channel.

Proposition I.1 *Under mixing and asymptotic mean stationarity conditions on the state process $\{Q_i\}$, the capacity of the finite buffer channel is given by,*

$$C = \lim_{n\to\infty} C_n = C_0 \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \mathbb{P}\{Q_i \neq B\}, \tag{1}$$

where B denotes the full buffer state and C_0 is the capacity of the DMC. Furthermore, capacity can be achieved by an i.i.d. input process $\{X_i\}$.

This capacity is expressed in bits per packet. This can be translated to a transmission rate (bits/second) by taking into account the packet arrival process, based on some ergodic conditions on the arrival process. Note that the average packet arrival rate can be chosen to maximize this transmission rate.

We also studied the case where there is feedback available from the channel output to the transmitter, delayed by at least one symbol. We showed that feedback in this case does not improve the channel capacity even though the channel could have complicated memory[2].

Finally we study a model of transmission in the absence of sequence numbers on the packets. This can be studied as a deletion channel. Similar problems have arisen in the context of transmission in the presence of synchronization errors, studied in [4] among others. This is a difficult problem in general and we study specific deletion models and develop some bounds for achievable performance.

References

[1] Venkat Anantharam and Sergio Verdú, "Bits through Queues," *IEEE Transactions on Information Theory*, vol. 42, number 1, pp. 4–18, January 1996.

[2] Robert G. Gallager, *Information theory and reliable communications*, John Wiley and Sons, New York, 1968.

[3] Robert M. Gray, *Entropy and Information Theory*, Springer-Verlag, New York, 1990.

[4] V I. Levenshtein, "Binary codes capable of correcting deletions, insertions and reversals," *Soviet Physics - Doklady*, vol. 10, number 8, pp. 707–710, February 1966.

[1]This is conjectured due to the result in [1] that coding in interarrival times is unnecessary when the alphabet size of the transmitted symbol is large (packet sizes in current networks range from a few tens of bytes to a few thousand). Though this was proved in the context of infinite buffer channels, we believe that this is true in our case as well.

[2]Note that in our model, feedback is only used for channel coding; we assume that the packet arrival process does not depend on feedback. If we remove this assumption, we are in the realm of congestion control, and schemes can be developed that achieve higher throughput. We have not addressed that problem here.