Even One-Dimensional Mobility Increases Ad Hoc Wireless Capacity

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I. Summary

The study of the capacity of wireless ad hoc networks has received significant attention recently. Gupta and Kumar [1] considered a model in which n nodes are randomly located in a disk of unit area and each node has a random destination node it wants to communicates to. They showed that as the number of nodes n increases, the throughput per source and destination (S-D) pair goes to zero like $1/\sqrt{n}$ even allowing for optimal scheduling and relaying of packets. The nodes are however assumed to be fixed. Grossglauser and Tse [2] considered an alternative model in which the nodes are mobile, and they showed that in sharp contrast to the fixed node case, the throughput per S-D pair can actually be kept constant even as the number of nodes scales. This performance gain is obtained through a *multiuser diversity* effect.

In the mobility model considered in [2], the trajectory of each node i is an independent, stationary and ergodic random process $X_i(t)$ with a uniform stationary distribution on the unit disk. Intuitively, this implies that a sample path of each node "fills the space over time". This mobility model is unrealistic in many practical settings. A natural question that arises is then how strongly the throughput result in [2] depends on this mobility model.

We show in this paper that the throughput result in [2] still holds even when nodes have much more limited mobility patterns. Specifically, we consider a model in which each node i is constrained to move on a single-dimensional great circle G_i on the unit sphere. Each node moves randomly along its own circle. The throughput capacity of such a network of course depends on the configuration of the great circles. Our main result is that if the locations of the great circles are chosen randomly and independently, then for almost all configurations of such great circles, the throughput per S-D pair can be kept constant as the number of nodes increases. Thus, although each node is restricted to move in a one-dimensional space, the same asymptotic performance is achieved as in the case when they can move in the entire 2-D region.[2]

II. Problem Statement and Main Result

The *constellation* $C = \{G_1, \ldots, G_n\}$ is a set of n great circles on the unit sphere. Each great circle G_i is randomly placed independently and uniformly on the sphere, but once generated, the constellaion C remains fixed for all time. The node location $X_i(t)$ is constrained to its great circle G_i, but is random (stationary and ergodic with a uniform stationary distribution) on it. Each node acts as the source of an infinite stream of packets with a random, but fixed destination D. As in [2], we assume that a packet can be successfully transmitted if the SIR at the receiver is above a given threshold [3].

There were two main reasons why the result in [2] yielded an asymptotically constant throughput.

Property I Every node spends the same order of time as the nearest neighbor to every other node. This ensures that each source can spread its packets uniformly across all other nodes, all acting as relays, and these packets can in turn be merged back into their respective final destinations.

Property II When communicating with the nearest neighbor receiver, the capture probability is not vanishingly small in a large system, even though there are $O(n)$ interfering nodes transmitting simultaneously.

The second property ensures that a high *aggregate* throughput (i.e. $O(1)$) of packets can be transmitted from the source to its nearest neighbors. The first property ensures that this aggregate throughput can be distributed evenly among all the relay nodes and there are no bottlenecks/hotspots in the relaying mechanism. It ensures that if we think of the different relays as queues for a given S-D pair, there is an equitable distribution of the traffic over these queues.

In the one-dimensional mobility model considered here, the symmetry among users is absent. Therefore, it is a priori unclear if the two properties can be satisfied, because once the constellations are fixed, (a) some nodes have a greater proclivity to be near particular nodes, resulting in a non-uniform spread of traffic, and (b) there can be areas of concentration, making the probability of successful transmission diminishingly small. It is in fact not true that the above policy can achieve a constant throughput for *every possible* configuration, but only for most. Our main result is as follows.

Theorem II.1 *There exists a relaying policy and a $\lambda > 0$ such that for almost all constellations as $n \to \infty$, a throughput of λ can be achieved for every source-destination pair, i.e., the probability of the set of constellations for which the policy achieves a throughput of λ goes to 1 as $n \to \infty$.*

Proof idea: The key ingredient in the proof is an appropriate definition of "typical" constellation, and showing that typical constellations occur with high probability for large system sizes. Basically, we require that there cannot be any disk on the sphere through which there is an atypically large number of great circles passing through. More precisely, for a given constellation C, define $M_C(x, r)$ to be the number of great circles passing through a disk of radius r around x. Let $\bar{M}(x, r)$ be the expected number of such great circles in a random constellation. A constellation C is said to be *ϵ-typical* if

$$\frac{|M_C(x,r) - \bar{M}(x,r)|}{n} < \epsilon \qquad \text{for every point } x \text{ and every } r > 0,$$

i.e. the empirical fraction passing through each disk is close to the expected value. The theory of uniform convergence of empirical measures [3] can be invoked to yield a bound on the probability that a constellation is atypical. In our problem, the class of sets on which the empirical measure is evaluated is the class of all disks $\mathcal{D}$. The key parameter in the bound is the Vapnik-Chervonenkis dimension $VC_d(\mathcal{D})$of this class, and this has been shown to be finite in [1].

References

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[3] Luc Devroye and Laszlo Gyorfi and Gabor Lugosi, *A Probabilistic Theory of Pattern Recognition*, Springer Verlag, New York, 1996.

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[2]We focus on the unit sphere instead of the unit disk to avoid edge effects, but we believe that the results should also be valid for nodes moving along line segments within the unit disk.

[3]Here, we use spherical distance.